Parent Speak Long Term Plan Year 9

Mathematics is the music of reason

| **Half Term** | **Unit Title** | **Key Knowledge/Content to learn and retain** | **Essential Skills to** **acquire (subject & generic)** | **Link to intent and ethos** | **Anticipated misconceptions** | **Links to earlier KS** | **Link to future KS** | **Opportunity for stretch and high prior attainers** | **Cultural Capital** | **Career Link** |
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| AUT 1 | **Unit 1: Number** | ***F1.1*** Calculations | **Apply systematic listing strategies**.  Use priority of operations with positive and negative numbers.  Simplifying calculations by cancelling.  Use inverse operations. | **Using Mathematics: Real Life Applications**    Everyone uses numbers on a daily basis often without really thinking about them. Shopping, cooking, working out bills applying for transport and measuring all rely on good understand of numbers | Writing (6 × 4) ÷ 2 as  6/2 × 4/2 = 3 × 2 = 6 | This topic provides a good opportunity to revisit and practise arithmetic skills using both mental and written methods including formal algorithms. For Foundation students this may include reviewing the use of number lines and the extended number system beyond the natural numbers for working with directed numbers. For an additional game to practise basic use of ‘BODMAS’, try NRICH ‘The 24 Game’. For students struggling with the concept of negative numbers, revisiting a couple of KS3 problems on the NRICH website could be useful: NRICH ‘Strange Bank Account’ and ‘Strange Bank Account Part 2’. (nrich.maths.org) | This is a straightforward topic at GCSE but to move successfully to KS5 confidence with calculations involving integers, order of operations and inverse operations will be required for algebraic manipulation and equation solving. This will be developed further to include other types of number, such as complex numbers, and modular arithmetic, which in turn links to group theory and congruence classes | **NRICH ‘Largest Product’**. This is a useful investigation that will ensure students become familiar with the terms ‘sum’ and ‘product’, which can be extended to non-integers; students could be encouraged to try to generalise their findings.  **NRICH ‘Cinema Problem’**. This is great for developing problem-solving skills and can be approached by exhaustive listings of calculations or introduction of algebra, so offers the possibility of extension for Higher tier students.  **NRICH ‘Up, Down, Flying Around’** balloon game. This is suitable for students who struggle with the concept of adding and subtracting negative integers.  **NRICH ‘Consecutive Negative Numbers’**. This investigation will ensure students have plenty of practice at adding and subtracting negative integers, while encouraging students to look for patterns and develop their investigation.  Extend the ‘Think of a number’ by asking students to write their own problems and give to another student to work through.  **NRICH ‘Twisting and Turning’**. Although this is marked as a Key Stage 3 activity this is a useful way to help students think about ‘undoing’, thus developing concepts such as inverse operations, inverse functions and in particular reciprocals since the ‘turn’ operation is to use a reciprocal action. (nrich.maths.org)  Twisting and Turning has two follow-on activities: NRICH ‘More Twisting and Turning’ and NRICH ‘All Tangled Up’, which further develop the use of reciprocals. | **Recommended Reading**    **The Music of the Primes by Marcus Du Sautoy**  Age 14+  How can one predict when the next prime number will occur? Is there a formula which could generate primes? These apparently simple questions have confounded mathematicians ever since the Ancient Greeks. In 1859, the brilliant German mathematician Bernhard Riemann put forward a hypothesis which finally seemed to reveal a magical harmony at work in the numerical landscape. The promise that these eternal, unchanging numbers would finally reveal their secret thrilled mathematicians around the world. Yet Riemann never publicly provided a proof for his hypothesis and his housekeeper burned most of his personal papers on his death. Whoever cracks Riemann's hypothesis will go down in history, for it has implications far beyond mathematics. In business, it plays a central role in security and e-commerce. In science, it brings together vastly different areas, with critical ramifications in Quantum Mechanics, Chaos Theory and the future of computing. Pioneers in each of these fields are racing to crack the code and a prize of $1 million has been offered to the winner. As yet, it remains unsolved.    **The Penguin Dictionary of Curious and Interesting Numbers by David Wells**  Age 14+  Look up 1729 to see why it is 'among the most famous of all numbers'. Look up 0.7404 (=π18) to discover that this is the density of closely packed identical spheres in what is believed by many mathematicians (though it was at that time an unproven hypothesis) and is known by all physicists and greengrocers to be the optimal packing. Look up Graham's number (the last one in the book), which is inconceivably big: even written as a tower of powers (999⋯) it would take up far more ink than could be made from all the atoms in the universe. It is an upper bound for a quantity in Ramsey theory whose actual value is believed to be about 6. A book to be dipped into at leisure.    **Things to Make and Do in the Fourth Dimension by Matt Parker**  Age 14+  This is the complete guide to exploring the fascinating world of maths you were never told about at school. Stand-up comedian and mathematician Matt Parker uses bizarre Klein Bottles, unimaginably small pizza slices, knots no one can untie and computers built from dominoes to reveal some of the most exotic and fascinating ideas in mathematics. Starting with simple numbers and algebra, this book goes on to deal with inconceivably big numbers in more dimensions than you ever knew existed. And always with something for you to make or do along the way  **Keywords**  **Function** - A special relationship where each input has a single output.  **Inverse** - Inverse means the opposite in effect. The reverse of.  **square root** - A square root of a number is a value that, when multiplied by itself, gives the number.  **cube root** - The cube root of a number is a special value that, when used in a multiplication three times, gives that number.  **Highest common factor (HCF)** - The greatest number that is a factor of all your chosen numbers  **lowest common multiple (LCM)** - The smallest positive number that is a multiple of two or more numbers  **Prime factor decomposition -** Finding which prime numbers multiply together to make the original number.  **Base** - The number that gets multiplied when using an exponent.  **Index** – The index of a number says how many times to use the number in a multiplication.  **Power** – The power (or exponent) of a number says how many times to use the number in a multiplication.  **Product** - The answer when two or more values are multiplied together.  **standard form** - A general term meaning "written down in the way most commonly accepted”.  **scientific notation** - Where a number is written in two parts: First: just the digits (with the decimal point placed after the first digit), Followed by: ×10 to a power that will put the decimal point back where it should be ordinary numbers.  **Surd** - A number that can't be simplified to remove a square root (or cube root etc)  **Rational Number** - A number that can be made as a fraction of two integers (an integer itself has no fractional part). |  |
| F1.2 Decimal numbers | Round to a given number of decimal places.  Multiply and divide decimal numbers.  Use pictures to help you solve problems. | **Using Mathematics: Real Life Applications**    Food technologists analyse the content of different raw and prepared foods to work out what they contain and how much there is of each ingredient. For example, how much water, protein, and fat there is in a cut of meat. They use decimal fractions to give the quantities correct tot tenths, hundredths, or even smaller parts of a gram. | Omitting zeros after the decimal point when asked to round numbers to a given number of decimal places. | This topic puts into practise the techniques developed using the four rules with integers.  Ordering integers.  Place value. Identifying that the position of a digit in a number determines its ‘size’ is as important for digits to the right of the decimal point as well as for digits to the left. | In integration, the techniques can sometimes reproduce the original function being integrated, as if it is recurring.  There are methods for dealing with this type of function. | The Gelosia method of multiplying decimals can be used as a procedure or students can begin to explain why it  works. Search the internet for ‘Gelosia method’.  NRICH ‘Does This Sound about Right?’ introduces a series of statements for students to investigate by estimating the calculations. | **Food technologists**  **“***The laws about labelling food are fairly strict. Manufacturers need to state exactly what is in their product and give exact amounts of different ingredients, so I have to measure things very accurately.”*  Food scientists and food technologists develop food and drink products, making sure they are safe to consume.  **Salary: £20,000 to £45,000**  **Sector**: Hospitality and food Manufacturing Science and research |
| F1.3 Place value | Convert metric measures.  Write decimal numbers of millions.  Round to a given number of significant figures.  **Estimate answers to calculations.**  **Use one calculation to find the answer to another.** | **Using Mathematics: Real Life Applications**    When you read that 34 000 people attended a festival, the actual number is likely to be slightly less or slightly more than that. When you roughly estimate what your spent over the weekend, look at an object and guess it is about 2.5 meters long or say things like, ‘I live about 15 kilometres from school’ you are estimating and using approximate values. | Have a strong understanding of place value.  Be confident with using decimals, particularly involving division.  Be able to make estimates for calculations to decide if a solution is reasonable. To understand inequality notation (to help understanding of bounds). | Have a strong understanding of place value.  Be confident with using decimals, particularly involving division.  Be able to make estimates for calculations to decide if a solution is reasonable. | Estimations and approximation will continue to have general applications within a range of calculation types through  A level, both in Mathematics and any subjects where calculations are required. There are many cross-curricular links  where accuracy is important e.g. calculating concentrations in Chemistry, or use of formula within Business Studies  or Economics. Within the Mathematics content, they have specific application in the Taylor and MacLaurin series and  solutions by iteration. | How many potential examples of rounding can students find in today’s news headlines?  NRICH ‘Does This Sound about Right?’. A good activity to ensure students are able to identify realistic estimations in context. |  |
| H1.2 Place value and estimating | **Estimate an answer.**  **Use place value to answer questions**. | Students may struggle to recognise that it is more appropriate,  for example, to round 62.1 to 64 than 60, if they are subsequently required to take the square root. |
| F1.4 Factors and multiples | Recognise 2-digit prime numbers.  Find factors and multiples of numbers.  Find common factors and common multiples of two numbers.  **Find the HCF and LCM of two numbers by listing.** | **Using Mathematics: Real Life Applications**    People use numbers and basic calculations daily. A market stall holder must quickly calculate the cost of a customer’s order, a logistics manager has to order stock and divide the supplies so that they are never over- or under-stocked. There are many applications of basic calculations | Confusing HCF and LCM. | Students should be able to multiply and divide Integers.  Indices: Students should understand index notation and be able to express a series of repeated multiplications in this way. | Indices.  The fundamental theorem of arithmetic | **NRICH ‘Factors and Multiples Puzzle’**. A challenging puzzle that asks students to arrange numbers in a table based on different headings, e.g., multiples of three. To differentiate this activity the teacher could place the headings on the table before giving to the students.  **NRICH ‘Sieve of Eratosthenes’.** This is an excellent way of students identifying prime numbers. This activity also helps students to understand what a prime number is.  **NRICH ‘How Much Can We Spend?’** An activity based on lower common multiples.  **NRICH ‘Take Three from Five’** is an investigation that challenges students to find a set of five numbers, from which it is not possible to select a set of 3 numbers that sum to a multiple of 3.  Students get pieces of ‘incorrect homework’. They need to identify the mistakes and correct the homework.  Show students how to apply prime decomposition to be able to easily simplify fractions and hence use this as a ‘trick’ for division. Ask them to create ‘difficult’ division questions for each other to solve (such as 1512 ÷ 54) where, thanks to prime decomposition, they know there must be an integer solution.  **NRICH ‘Factoring a Million’.**  **NRICH ‘Gaxinta’.** | **Logistics Manager**  *“Counting in multiples saves quite a bit of time. If I know that each shelf has 15 boxes and each box contains 5 reams of paper, the I know straightaway that I have 75 Reams on each shelf without having to count each ream*.”  Supply chain managers organise the movement of goods and materials from suppliers and manufacturers to customers.  **Salary: £24,000 to £48,000**  **Sector**: Delivery and storage Managerial Transport |
| F1.7 Prime factors | **Write a number as the product of its prime factors.**  **Use prime factor decomposition and Venn diagrams to find the HCF and LCM.** | Writing addition signs or commas in place of multiplications signs when writing a number as a product of its prime factors. |
| H1.3 HCF and LCM | **Write a number as the product of its prime factors.**  **Find the HCF and LCM of two numbers.** | Students might confuse the HCF with the LCM, especially when calculating from a Venn diagram. |
| F1.5 Squares, cubes and roots | Find square roots and cube roots.  Recognise powers of 2, 3, 4 and 5.  **Understand surd notation on a calculator.** | **Using Mathematics: Real Life Applications**    Interior designers use square units to work out the area of floors to be tiled and walls to be painted. They then work out how much paint to buy and use the size of the tiles (also in quare units) to work out how many are needed | Forgetting to apply the priority of operations when working out calculations involving fractions. | From Key Stage 3 students should be confident with square and cube numbers, and know the first 15 squares, six cubes and their corresponding roots.  Geometrically students should recognise the link between squaring a number and finding the area of a square from its length. Similarly they should make the link between the volume of a cube and its edge lengths. | Students will learn to differentiate and integrate terms that involve indices and they will develop the laws of indices to work with logarithms, which are used extensively at A level and beyond, for example linking to e, infinite series and solving differential equations. Indices are present in topics such as geometric series and binomial distribution, and the ability to manipulate expressions involving indices is an important algebraic technique. | The problem ‘**Power Mad’** from NRICH gets students to investigate a range of patterns in powers.  **NRICH ‘A Biggy’**. A problem that will stretch students to think more deeply about the properties of numbers that have been raised to powers  **NRICH ‘Negative Power’** is an interesting investigative problem involving negative indices and raising a power to a power.  **(Higher only)** The ‘RISPs Task 35 Index Triples’ is a challenge that extends toward A level that investigates the effect of the order when writing numbers to the power of each other. To access this task, go to the **RISPs homepage** and then select ‘Risps 1 – 40’. | **Interior designer**  *“I’m pretty good at estimating in quare units. I can usually look at a room and guess the area of the floor and walls quite accurately. Tiles are harder, I do rough sketches on squared paper to help me work out how many tiles of a particular size are needed to cover a floor area”*  Interior designers plan and supervise the layout and decoration of the inside of buildings.  **Salary: £18,000 to £45,000**  **Sector**: Creative and media |
| F1.6 Index notation | Use index notation for powers of 10.  Use index notation in calculations.  **Use the laws of indices.** | Use correct mathematical language and notation |
| H1.4 Calculating with powers (indices) | **Use powers and roots in calculations.**  **Multiply and divide using index laws.**  **Work out a power raised to a power.** | Students will struggle to identify the common base |
| H1.5 Zero, negative and fractional indices | **Use negative indices.**  **Use fractional indices.** | With questions like 36^(3/2) students might try to multiply 36 by 3/2 or just be very unsure about how to start the question |
| H1.6 Powers of 10 and standard form | **Write a number in standard form.**  **Calculate with numbers in standard form.** | **Using Mathematics: Real Life Applications**    The study of stars, moons and plants involves huge numbers. Astronomers use standard form to write or type very large quantities. This makes it easier for them to compare the quantities and it allows them to calculate with and without calculators. The Sun has a mass of 1.988 x 1030 kg. this is a number with 27 zeroes, and it would be clumsy and impractical to have to write it out each time you wanted to use it. | Students might think that, for example, 0.8 × 105 is in standard form.  When multiplying or dividing numbers in standard form, students find it difficult to convert an answer like 36 × 107 into standard form.  When rationalising a denominator, in a question like 5/75, students will forget to simplify their final answer. | Place value and rounding through Key Stages 2 and 3.  Significant figures.  Index laws for multiplication and division. | Indices. These are reinforced early in the Core topics and are used extensively throughout the course.  Applications within physics, chemistry, and biology | Converting standard form to ordinary numbers. Research data for very large and small numbers such as the diameter of an atom, size of the smallest virus, number of atoms in a human body, diameter of the Earth  **NRICH ‘Big and Small Numbers in the Living World’**. Combining estimation and large numbers with problem solving skills to help contextualise the use of standard form. | **Astronomer**  *“In astronomy we work with very large and very small numbers. They are 100 000 000 000 000 000 000 000 known stars alone! Imagin having to write this numer out in full every time you wanted to use it! It is much easier to write 1 x 1023”*  Astronomers study the origin and structure of the universe, including its planets, stars, galaxies and black holes.  **Salary: £15,609 to £60,000**  **Sector**: Science and research |
| H1.7 Surds | **Understand the difference between rational and irrational numbers.**  **Simplify a surd.**  **Rationalise a denominator.** | **Using Mathematics: Real Life Applications**    Surds are only really used when you are doing mathematical calculations that require exact answers. Surds cannot really be used for practical purposes. You cannot tell a builder to cut a length of steel that is metres long, because is an irrational number; so you would be more likely to specify an approximate lengths of 1.41 metres | When rationalising a denominator, in a question like 5/75, students will forget to simplify their final answer. | Squaring numbers and finding square roots.  Using the laws of indices  Prime factor decomposition  Using Pythagoras’ theorem.  Cancelling fractions and equivalent fractions  Expanding brackets and simplifying expressions (basic algebraic manipulation skills).  The difference of two squares. (Students will be able to rationalise without knowing that they are using the factor pairs of the difference of two squares but, if they have previously met this topic, it is a good example of a technique being put into use in another context.) | Exact values of solutions are often asked for and these may need to be given in surd form.  Complex numbers use similar techniques to the rationalising of the denominator.  Binomial theorem (to approximate roots). | **NRICH ‘Trice’ is** a problem that can be solved using Pythagoras’ theorem and leaving intermediate answers in surd form will allow students to obtain a precise solution.  **NRICH ‘The Spider and the Fly’** also uses Pythagoras’ theorem and students could use surds but will need to convert them to approximate values to compare  **NRICH ‘The Root of the Problem’** will require students to practise rationalising binomial denominators.  As an extension for students finding manipulation of surds quite straightforward, the task **NRICH ‘Irrational Arithmagons’** should prove quite a challenge.  Research the ‘Geometric Square Root’ construction technique described by Descartes (although not attributed to him). Create a presentation or leaflet explaining how the method works.  For very able students a nice extension is to look at the formula for the Fibonacci sequence. | **Print works manager**  *“This widths and lengths of A-series rectangular paper were developed using the ration 1: to mathematically construct a rectangle of area 1m2 (A0 size). In real life paper is cut to exact millimetre size to A0 is 841 mm x 1189 mm rather than 841 whish is 1189.353606… mm.”*  Pre-press operators work in the printing industry, getting artwork from a client and supplying the printing plates used on a press.  **Salary: £16,000 to £30,000**  Sectors: Computing, technology and digital Creative and media |
| AUT 2 | **Unit 2: Algebra** | F2.1 Algebraic expressions | Use correct algebraic notation.  Write and simplify expressions. | **Using Mathematics: Real Life Applications**    Algebra lets you describe and represent patterns using concise mathematical language.  This is useful in many different career including accounting, navigation, building, plumbing, health, medicine, science and computing | Students may write terms with letter before number and writing answers in the wrong alphabetical order.  Some may not realise that x2 and x are not like terms.  1a = a’ is not always obvious to students  Students may not write the answer as a fraction when terms are divided.  Mistakes with negatives are common. | There are a few prior connections for this topic  The order in which operations are completed (BODMAS). Opportunities for revising these rules come through substitution and remembering that for 2x2 , x is squared before it is multiplied by 2. | At A level, the demand on students’ algebraic manipulation skills is high, particularly when working with fractions and solving equations. Their knowledge is extended beyond laws of indices to include laws of logarithms and students are expected to be able to manipulate expressions with ease. You may observe that students with weak conceptual understanding of the use of algebra and functions in mathematics struggle greatly with the A level course and will happily cancel a variety of things when simplifying or expanding brackets incorrectly This prevents them working with a variety of new material as they can’t start the problem correctly and limits their ability to be successful in the course. Having a strong conceptual understanding regarding variables and the difference between an identity and an equation will be important when students prove identities in trigonometry | **NRICH ‘Crossed Ends’.** There are many investigations that can support students’ development in forming appropriate expressions to generalise their findings. This one is effective because it uses only a 100-square grid. When students have worked on expanding two brackets they can also look at a similar problem using a 100-square grid where they compare the product of the opposite corners of a square drawn on the grid. There are many shapes that make interesting number problems on a 100-square grid. (nrich.maths.org)  Toy train sets can be used to introduce the idea of simplifying where the variable is the length of the track piece. Given that there are only a few different types of track pieces, a track can be made up using two or more pieces and its total length represented by an algebraic expression. (Note: you only need pictures of toy train tracks to do this task; having an actual set only enhances this task for weaker students.)  **NRICH ‘Perimeter Expressions’**. A similar, far more challenging NRICH task based on lengths of proportional sizes of paper (having only used cuts that cut a piece exactly in half). (nrich.maths.org)  It is very useful for developing students’ awareness of a variable as all paper sizes are proportional and students completing this task with different sizes of A paper can bring out some generality.  Setting up a worksheet that consists of three columns where the first column contains a series of single bracket expansions to be found and the other two are blank. Students fold over the paper to hide the right-hand column and students write the expansion of the factorised expressions in the middle column. The students unfold the last column and fold over the left-hand column. Students are required to then factorise the expanded form in the centre column. This task is very good for helping students identify the difference between factorised and fully factorised form and they remember fewer of the original questions than you’d think. | **Recommended Reading**    **Blockhead: The Life of Fibonacci**  Age 7+  As a young boy in medieval Italy, Leonardo Fibonacci thought about numbers day and night. He was such a daydreamer that people called him a blockhead.  When Leonardo grew up and travelled the world, he was inspired by the numbers used in different countries. Then he realized that many things in nature, from the number of petals on a flower to the spiral of a nautilus shell, seem to follow a certain pattern. The boy who was once teased for being a    **The Math Book by Clifford A Pickover**  Age 12+  Mathematic's infinite mysteries and beauty unfold in this book. Beginning millions of years ago with ancient ant odometers and moving through time to our modern-day quest for new dimensions, prolific polymath Clifford Pickover covers 250 milestones in mathematical history. Among the numerous concepts readers will encounter as they dip into this inviting anthology: cicada-generated prime numbers, magic squares, the discovery of pi and calculus, and the butterfly effect. Each topic is presented in a lavishly illustrated spread, including formulas, fascinating facts about scientists' lives and real-world applications of the theorems.  **Keywords**  **Term** – In Algebra a term is either a single number or variable, or numbers and variables multiplied together.  **like terms** – Terms whose variables (such as x or y) with any exponents (such as the 2 in x2) are the same.  **Variable** - A symbol for a value we don't know yet. It is usually a letter like x or y.  Identity - An equation that is true no matter what values are chosen.  **Equation - An equation says that two things are equal.**  **Expand** - Expand is when we multiply to remove the ( )  **consecutive integers** - Numbers which follow each other in order, without gaps, from smallest to largest.  **Common factor** – “Factors" are numbers we can multiply together to get another number. When we find the factors of two or more numbers, and then find some factors are the same ("common"), then they are the "common factors".  **Highest common factor** (HCF) - The greatest number that is a factor of all your chosen numbers  Formula - A rule or fact written with mathematical symbols.  **Expression** - Numbers, symbols and operators (such as + and ×) grouped together that show the value of something  **Equation** - An equation says that two things are equal.  **Identity -** An equation that is true no matter what values are chosen.  **Formula -** A rule or fact written with mathematical symbols.  **Variables** - A symbol for a value we don't know yet. It is usually a letter like x or y.  **Arithmetic sequence** - A sequence made by adding the same value each time.  **common difference** - The difference between each number in an arithmetic sequence  **Geometric sequence** - A sequence made by multiplying by the same value each time | **Games designer**  *“You are unlikely to think about algebra when you watch cartoons or play video games, but animators use complex algebra to program the characters and make objects move”*  Computer game developers create video games for phones, tablets, PCs and consoles.  **Salary: £25,000 to £70,000**  **Sector**: Computing, technology and digital |
| F2.2 Simplifying expressions | Use the index laws.  **Multiply and divide expressions.** | Not realising that algebraic terms must be ordered correctly  Not realising that 1/2 x = x / 2  Thing that x / x = 0 |
| H2.1 Algebraic indices | Use the rules of indices to simplify algebraic expressions. | Students may think that negative indices involve multiplying by negative numbers (e.g. that a–2 = –2a) and that fractional indices involve division |
| F2.3 Substitution | Substitute numbers into expressions.  Write more complex expressions.. | Mistakes with priority of operations are common, such as not calculating indices first, and ignoring or not correctly dealing with a negative outside a bracket. |
| F2.5 Expanding brackets | Expand brackets.  **Simplify expressions with brackets.**  Write and use formulae with brackets. | Students often forget to take note of a negative sign when multiplying brackets.  Students often forget to multiply the term outside the bracket by the last term in the bracket.  Students often forget to include brackets when writing expressions that need them. |
| F2.6 Factorising | **Factorise algebraic expressions.**  **Use the identity symbol ≡ and the not equal to symbol ≠.** | Failing to factorise an expression completely.  When the HCF is a term in the expression, students can incorrectly think that it ‘disappears’ after factorisation. |
| H2.2 Expanding and factorising | E**xpand brackets.**  **Factorise Quadratic expressions.** | Students may use signs incorrectly or forget to include them in calculations, particularly when there is a negative number both outside and inside a bracket. |
| H2.3 Equations | **Solve equations involving brackets and numerical fractions.**  **Use equations to solve problems** | **Using Mathematics: Real Life Applications**    Accounting involved a great deal of mathematics. Accountants set up computer spreadsheets to calculate and analyse data. Programs such as Microsoft excel work by applying different equations to values in columns or cells , so you need to know what equations or formulae to use to get the result you need | Students may not multiply all the terms in the numerator of a fraction | This chapter uses all the manipulation skills worked on in in key stage 3 algebra to help manipulate expressions including factorising and simplifying to solve a variety of equations | Solving forms a large part of the future Mathematics students will study. Students will learn how to manipulate equations containing a larger range of functions to find multiple solutions and have a greater understanding about the number of solutions in a given range.. The ideas surrounding inverses will also be covered in greater detail with students learning about the domain and range of functions as well as the restrictions on these that must sometimes be in place for an inverse to exist. | **NRICH ‘Good Work If You Can Get It’** is a challenging problem in terms of forming and solving. There is a lot of information to get through and students will need to define their own variables for the amount given to each man and the total amount. The trick to solving comes from knowing the fraction of the total amount that each man is due based on the number of days he worked. | **Accountant**  *“Although the computer does the actual calculations, I must insert different equations to tell it what operations to perform and in which order to perform them. It is important to check that the equations are production the correct answers, though.”*  Management accountants look after a company's finances and find ways to improve profitability.  **Salary: £22,000 to £55,000**  **Sector**: Business and finance Managerial |
| F2.4 Formulae | Recognise the difference between a formula and an expression.  Write and use formulae.  Use smaller numbers to help you see a pattern. | **Using Mathematics: Real Life Applications**    Vets use formulae to make sure they are giving animals the correct dosage of medicine for their age and mass. A poodle weighing 6 kg needs a far smaller does of medicine than a 35kg retriever | Address the misconception that 32 is the same as 2 × 3 or 43 is the same as 3 × 4, etc. | Since this is a chapter on formulae it has connections with all areas of mathematics where you are required to calculate an answer given several inputs. During this chapter your students will be revisiting the process of calculating areas, perimeters, volumes, solutions to quadratic equations (and looking forward to using trigonometric functions,  Pythagoras’ theorem and equations of straight-line graphs). | Students looking further ahead will need to be fluent in their use of all the formulae mentioned in this chapter, particularly those relating to equations of motion and calculus topics. A level students will be required to rearrange formulae with the variable appearing several times, for example, rearrange a formula containing a quadratic term, say x2 , to make x the subject. Further to this, recurrence relations will be tied together with different types of sequences where students will be required to recognise more than one way of rearranging the subject to give a recurrence relation to employ Numerical Methods. | The activity NRICH **‘How Do You React?’** could link this section with Interpretation of graphs. The students are required to come up with their own formula for this situation  Links can be made forward with the activity NRICH **‘Making Maths: Make a Pendulum’** and an investigation, where you drop an object from varying heights and find the time taken to fall. This data can then be plotted and students could come up with a formula to describe this situation | **Veterinary surgeon**  *“I need to make sure I give the animals I treat the correct amount of medicine. I do this by using formulae that take into account their age, mass and the ratio between prescribed medicine”*  Vets diagnose and treat sick or injured animals.  **Salary: £30,000 to £50,000**  **Sector**: Science and research Animal care |
| F2.7 Using expressions and formulae | Write expressions and simple formulae.  Use maths and science formulae. | Students may not always remember to use the correct priority of operations.  Many students find it difficult to work through the language of the question to understand what is being asked of them, so they will require prompting.. |
| H4.2 Formulae | **Expand the product of two brackets.**  **Use the difference of two squares.**  **Factorise quadratics expressions of the form ‘𝑥^(2)’ + 𝑏𝑥 + 𝑐.** | Students may write down only the right-hand side of a formula  Some students may not use the priority of operations when evaluating formulae. |
| H2.5 Linear sequences | **Find the general term or 𝑛th term of an arithmetic sequence.**  **Determine whether a particular number is a term of a given arithmetic sequence.** | **Using Mathematics: Real Life Applications**    Finding a pattern and working out how the parts of the pattern fit together is important in scientific discovery. Scientists use sequences to model and solve real- life problems, such as estimating how quickly disease spread. | Students may think the first term of a sequence is u0 not u1.  Some may misunderstand what they have found when solving an equation for the nth term to find n. | This topic provides a good opportunity to return to work on basic calculations and properties of numbers. There are opportunities to consider how square and cube numbers are related to physical shapes and how a numerical sequence can be linked to geometrical or physical patterns. | This topic will be built upon in KS5. Having a strong understanding of this concept will be necessary for students to extend their knowledge in A level modules. In addition to the notation learnt at GCSE, students will also learn to describe sequences as arithmetic progressions. They will calculate terms in a sequence and the sum to n terms.  In addition to arithmetic progressions, geometric progressions are a further extension of this topic, which builds upon sequences such as 2, 4, 8, 16, 32. Students will learn to calculate terms in a sequence and the sum to n terms, including the sum to infinity for converging sequences. | **NRICH ‘Seven Squares’ task**. The teacher notes with this resource suggest how to use this in the classroom, enabling the formulation of the position-to-term rule to naturally emerge from discussions on the construction of the sequence. (nrich.maths.org)  Search for any of the ‘Zeno’s Paradox’ videos on the internet  **NRICH ‘Fibs’ task.** (nrich.maths.org)  **NRICH ‘Steel Cables’** task. (nrich.maths.org)  **NRICH ‘Handshakes’** task. (nrich.maths.org)  **NRICH ‘Mystic Rose’** is a different form of the ‘Handshakes’ problem. (nrich.maths.org) | **Medical researcher**  *“When a new outbreak of a disease ooccurs,I need to work out how quickly it is spreading. To do this I look at the sequence in which the numbers of victims are increasing. I use the sequence to predict how many people will become infected in a certain length of time.”*  Research scientists plan and lead experiments and investigations on a range of scientific topics.  **Salary: £17,688 to £43,000**  **Sector**: Environment and land Science and research |
| H2.6 Non-linear sequences | **Solve problems using geometric sequences.**  **Work out terms in Fibonacci sequences.**  **Find the 𝑛th term of a quadratic sequence.** | Students may forget to divide the second differences by 2 to find the coefficient of n2 in the nth term formula for a quadratic sequence.  A common error is to subtract the given sequence from the sequence an2 when finding an nth term of the form un = an2 + bn + c. |
| H2.7 More expanding and factorising | **Expand the product of two brackets.**  **Use the difference of two squares.**  **Factorise quadratics expressions of the form ‘𝑥^(2)’ + 𝑏𝑥 +** 𝑐. | **Using Mathematics: Real Life Applications**    Situations that involve motion, including acceleration, stopping distance, velocity and distance travels (displacement) can be modelled using quadratic expressions and formulae. | Some students may write (a + b)2 = a2 + b2  Some students may neglect to deal with the product of negative numbers correctly. | The connections back to Algebra, are obvious as these ideas are just extended to quadratics in this chapter. However, more opportunities for generalising students’ long multiplication methods are offered here and students are better equipped to consider more challenging ‘I think of a number…’ problems.  There are several problems that use the knowledge of properties of shapes and formulae for calculating area that students will have met in Area. | At A level these manipulation skills are essential. Students will be required to manipulate quadratics with ease between various forms and understand how these different forms connect to features of a quadratic’s graph and their knowledge of transformations. They will also learn how to long divide polynomials in order to factorise expressions of higher degree than two  Students who work with grid expansion methods can adapt their grids to divide. Those that don’t have a conceptual understanding of how the grids support multiplication, and hence division, often struggle to learn a long new algorithm, both in traditional format and employing the use of a grid. A fluency of expansion of two or more binomials, trinomials and larger expressions will be required. | **‘Finding Factors’** from NRICH is a nice problem that could be used as an introduction to factorising quadratic expressions or to develop fluency in this skill. Once the pupils have played with this idea, they may want to create their own grids to give to another student to try. (nrich.maths.org)  NRICH has a collection of **short problems** that make use of expanding and factorising quadratics. They could be used individually or made into a race for pairs of students to solve. (nrich.maths.org | **Police road accident investigator**  *“At the site of a rach, I measure the lengths of the tyre skid marks and apply and equations to work out the speed at which vehicles were moving before the accident”*  Forensic collision investigators use science and engineering to investigate the causes of road traffic and vehicle related incidents.  **Salary: £20,000 to £40,000**  **Sector**: Transport Government services Emergency and uniform services |
| SPR 1 | **Unit 3 Interpreting and Representing Data** | F3.1 - Frequency tables | Designing tables and data collection sheets.  Reading data from tables. | **Using Mathematics: Real Life Applications**    We live in a very information-rich world. Knowing how to construct accurate graphs and how to interpret the graphs we see is important. Many graphs in print and other media are carefully designed to influence what we think by displaying data in particular ways. | Every data point needs a class interval in a data collection sheet.  Including 19 and/or 21 in the following class interval 19 < t < 21. e. | Addition of numbers.  Counting tally symbols | Histograms at A level introduce added complexities. Firstly, end points may be less obvious, and students may need to consider bounds. Secondly, the area of the bars becomes proportional to frequency rather than equal to it.  The idea of area below a curve standing for (or being proportional to) frequency becomes increasingly important as students begin considering distributions such as the normal. | More interpretation questions would be useful, including asking pupils to work out averages, range, percentage proportion or probability from data presented in a bar chart or pictogram.  As an extension activity, students could devise a method to draw comparative pie charts (for two data sets) where the area is proportional to the size of the population. Doubling the population does not double the radius…  **NRICH ‘What’s the Weather Like?’** provides data for three different UK towns over the last 50 years and suggestions for suitable research questions. (nrich.maths.org) This data was taken from the **Met Office website**, so this investigation could easily be adapted to your local area. (metoffice.gov.uk) | **Recommend Reading**    **Aha! Insight & aha! Gotcha by Martin Gardner**  Age 11+  Previously published separately, the two books aha! Gotcha and aha! Insight have been combined as a single volume. The aha! books, as they are referred to by fans of Martin Gardner, contain 144 wonderful puzzles from the reigning king of recreational mathematics. In this combined volume, you will find puzzles ranging over geometry, logic, probability, statistics, number, time, combinatorics, and word play. Gardner calls these puzzles aha! problems, that 'seem difficult, and indeed are difficult if you go about trying to solve them in traditional ways. But if you can free your mind from standard problem-solving techniques, you may be receptive to an aha! reaction that leads immediately to a solution. Don't be discouraged if, at first, you have difficulty with these problems. After a while you will begin to catch the spirit of offbeat, nonlinear thinking, and you may be surprised to find your aha! ability improving.'  **Keywords**  **Inequality** - An inequality compares two values, showing if one is less than, greater than, or simply not equal to another value.  **discrete data** - Data that can only take certain values.  **continuous data** - Data that can take any value (within a range).  **bar chart** – A graph drawn using rectangular bars to show how large each value is.  **Histogram** – A graphical display where the data is grouped into ranges (such as "100 to 149", "150 to 199", etc), and then plotted as bars.  **line graph** – A graph with points connected by lines to show how something changes in value: as time goes by, or as something else changes.  **Trend -** A line on a graph showing the general direction that a group of points seem to follow  **Circles** - A 2-dimensional shape made by drawing a curve that is always the same distance from a center.  **Fractions** – How many parts of a whole  **Percentages** - Parts per 100  frequency polygon - A graph made by joining the middle-top points of the columns of a frequency histogram.  **Correlation** – When two sets of data are strongly linked together we say they have a High Correlation.  **Variable** - A symbol for a value we don't know yet. It is usually a letter like x or y.  **Outlier** - A value that "lies outside" (is much smaller or larger than) most of the other values in a set of data.  **Bivariate data** - Data for two variables (usually two types of related data).  **scatter graph** - A graph of plotted points that show the relationship between two sets of data.  **Line of best fit** – A line on a graph showing the general direction that a group of points seem to follow.  **Interpolation** – Estimating a value inside a set of data points.  **Extrapolation** - Estimating a value outside a set of data points. | **Newspaper editor**  *“when we have data, wee need to display it so that our message has the maximum impact”*  Newspaper and magazine editors manage the style and content of printed publications.  **Salary: £25,000 to £50,000**  **Sector**: Creative and media |
| F3.3 - Representing data | Draw and interpret comparative and composite bar charts.  Interpret and compare data shown in bar charts, line graphs and histograms. | Students may not place bars in charts to highlight comparisons.  Using non-linear scales  Failing to correctly label the graph. | Determine what features are missing from a graph |
| **F3.2 - Two-way tables** | **Use data from tables.**  **Design and use two-way tables.** | Students may read timetables incorrectly. Mistakes are common when finding elapsed time between departure point and destination | Convert between 12- and 24-hour clock times.  Understand use of fractions. |
| **F3.4 - Time series** | **Plot and interpret time series graphs.**  **Use trends to predict what might happen in the future** | not all points have to follow the trend for the trend to be valid.  Points should be joined with straight lines.  understand the difference between ‘interpolate’ and ‘extrapolate’ | Placing smaller figures into larger figures. |
| **F3.5 - Stem and leaf diagrams** | **Construct and interpret stem and leaf and back-to-back stem and leaf diagrams.** | writing the leaf values on the left side of a back-to-back stem and leaf diagram incorrectly  not writing the key  Missing stem values | Place numbers in order of size. |
| **F3.6 - Pie charts** | **Draw and interpret pie charts**. | incorrectly use protractors to measure angles  Errors in equivalence of percentages and angles  Errors where the sums of angles do not equal 360° | Express a part of a circle as a fraction or percentage of the whole.  Know the number of degrees at the centre of a circle. |
| **Other Statistical diagrams (H3.1)** | **Construct and use frequency polygons** | Students may forget to read the left-hand side of a back-to-back stem and leaf diagram ‘backwards’: encourage students to read out from the stem, in both directions.  Some students may miss items of data when drawing stem and leaf diagrams; encourage students to make sure that the number of leaves they draw is the same as the number of items of data in the original list.  Students often find it difficult to interpret a pre-drawn frequency polygon, forgetting that the values are plotted at the mid-points of each class. This needs to be emphasised during the lesson. | The skills of graph drawing are called upon heavily here. | Standard deviation: a more sophisticated measure of spread (like range and IQR).  Linear regression: the process of creating an equation to represent a line of best fit that can be quickly used to make predictions. |
| **More Statistical diagrams (H3.6)** | **Choose proper diagrams to display data.**  **Recognise misleading graphs.** | **Using Mathematics: Real Life Applications**    Analysing large sets of data enables financial and insurance companies to make predictions about what might happen in the future. Car insurance premiums. Are worked out accordingly to typical or ‘average’ behaviour of large groups of people | Students can usually fill in the totals and missing numbers in two-way tables but may not be confident in what the intersection of a particular row and column mean.  Students may lose marks for giving insufficient reasons when asked to choose the best form of statistical diagram. It is often easier to explain why not to use a diagram rather than thinking of reasons to use it. | After the introduction of grouped frequency tables, this is a good opportunity to construct some cumulative frequency diagrams. Many students fall into the habit of plotting the cumulative frequency diagram at the midpoint of each class (like when estimating the median) rather than the end points.  The skills of graph drawing (Chapter 23F / 24H Straight-line graphs and Chapter 24F / 25H Graphs of functions and equations) are called upon heavily here. | Standard deviation: a more sophisticated measure of spread (like range and IQR).  Linear regression: the process of creating an equation to represent a line of best fit that can be quickly used to make predictions. | Students could look at recent news stories that report on data to see whether the type of average is specified (it rarely is!). They could then go to the source of the data, to find for themselves how the analysis was actually done.  Pick a future sporting event (it could be for a school team, football championship or an international event such as the Olympics). There is only one spot left on the team and the coach is unsure which player to choose. Give students the statistics for several players and tell them that, in a group, they need to use this data to present a case for one of the players. Each group has a different player to represent.  Students could seek out misleading graphs on the internet and present their favourite. The ‘junkcharts’ website is a good starting point.  Give the class a data set, e.g. traffic accidents over the past few years. They should aim to create a graph that overemphasises the trend using the techniques they have learnt  Make links with the science department. Is there an experiment that students have done recently that would utilise a scatter diagram with a linear line of best fit? If students have drawn scatter diagrams in science, they could be scanned and then peer- / self-assessment could be used in maths lessons. | **Insurance broker**  *“We group drivers together by age and gender and use statistics to find typical driving behaviour for each group. Young drivers have more accidents, so their insurance costs more”*  Insurance brokers use their knowledge of the insurance market to help find the right cover, at the best price, for their customers.  **Salary: £20,000 to £65,000**  **Sector**: Administration Business and finance |
| **F3.7 - Scatter graphs** | **Plot and interpret scatter graphs.**  **Decide whether there is a relationship between sets of data** | Believing that correlation can be set up from 2 or 3 pieces of data.  plotting an inaccurate linear scale |
| **F 3.8 - Line of best fit** | **Draw a line of best fit on a scatter graph.**  **Use the line of best fit to predict values.** | Students may incorrectly believe that a correlation can be established from 2 or 3 pieces of data.  It is a common error to plot an inaccurate linear scale.  It is common to make incorrect claims when extrapolating |
| **Averages and range** | **Decide which average is best for a set of data.**  **Estimate the mean and range from a grouped frequency table.**  **Find the modal class and the class holding the median.** | Students may find it difficult to decide whether or not to include outliers in the calculation of the range. The rule of thumb is to use all values unless it is clear that there is an error in the data set.  ● When calculating the mean, students sometimes divide by the total of the midpoint column (which is meaningless) instead of the frequency column. |
| SPR 2 | **Unit 4: Fractions and percentages** | F4.1 Working with fractions | Compare fractions.  Add and subtract fractions.  Use fractions to solve problems. | **Using Mathematics: Real Life Applications**    Nurses and other medical support staff work with fractions, decimals. Percentages, rates and ratios every day. They calculate medicine doses, convert between different systems of measurement and set the patient’s drips to supply the correct amount of fluid | Some students may incorrectly add the numerators and the denominators,  Some may not understand how to correctly work with equivalent fractions.  Some students may not use the LCD. | This topic provides many opportunities to practise the four rules with integers.  The order of performing operations, BODMAS, applies all the time, not just with integer values. | Vectors are a core and an applied topic at A level and often involve the use of fractions. Algebraic fractions come early in the A level course and are used frequently throughout, especially with some of the integration techniques | Odd one out. Give students a collection of fractions where all but one of them are equivalent. Students should justify their answer and explain it mathematically  fractions and music. Students can use the link between music and fractions to contextualise equivalent fractions. They can use the facts below to create their own clapping rhythm. For example, students could be given four bars to fill with notes. They can create their clapping rhythm using this idea ensuring that they have used all four bars.  The **‘Mostly Shape and Space Materials’** activity titled ‘Dissecting a Square S S3’ in Improving Learning in Mathematics is available from the National STEM Centre Archive. (nationalstemcentre.org.uk)  **NRICH ‘Fractions Jigsaw’**. This activity is an alternative way to practise the four rules with fractions rather than use a traditional textbook exercise. (nrich.maths.org)  **NRICH ‘Peaches Today, Peaches Tomorrow…**’. An excellent activity to practise finding fractions of amounts. (nrich.maths.org)  Music and fractions. Students can create their own musical instruments and practise multiplying fractions at the same time. Students will need eight glass containers that are the same size. Students will need to calculate how  much liquid to put into each container based on the fractions below. Students could then play simple music with their creations!  **NRICH ‘Ben’s Game’** is an excellent fractions-based activity that requires student to record their working effectively. It also covers factors and multiples as well as calculating with fractions. (nrich.maths.org) | **Recommended Reading**    **Mathematics for the Curious by Peter Higgins**  Age 13+  When do the hands of a clock coincide? How likely is it that two children in the same class will share a birthday? How do we calculate the volume of a doughnut? Mathematics for the Curious provides anyone interested in mathematics with a simple and entertaining account of what it can do. Author Peter Higgins gives clear explanations of the more mysterious features of childhood mathematics as well as novelties and connections that prove that mathematics can be enjoyable and full of surprises. Topics include: the truth about fractions, ten questions and their answers, and the golden ratio. Higgins poses entertaining puzzles and questions tempting the reader to ponder math problems with imagination instead of dread. Mathematics for the Curious is an accessible introduction to basic mathematics for beginning students and a lively refresher for adults.  **What the Numbers Say: A Field Guide to Mastering Our Numerical World by [Derrick Niederman, David Boyum]**  **What the Numbers Say: A Field Guide to Mastering Our Numerical World**  **by Derrick Niederman**  Ages 17+  A decade ago, computer scientist Douglas Hofstadter coined the term innumeracy, which aptly described the widespread ailment of poor quantitative thinking in American society. So, in What the Numbers Say, Derrick Niederman and David Boyum present clear and comprehensible methods to help us process and calculate our way through the world of “data smog” that we live in. Avoiding abstruse formulations and equations, Niederman and Boyum anchor their presentations in the real world by covering a particular quantitative idea in relation to a context–like probability in the stock market or interest-rate percentages. And while this information is useful toward helping us to be more financially adept, What the Numbers Say is not merely about money. We learn why there were such dramatic polling swings in the 2000 U.S. presidential election and why the system of scoring for women’s figure skating was so controversial in the 2002 Winter Olympics, showing us that good quantitative thinking skills are not only practical but fun.  **Keywords**  **Denominator** - The bottom number in a fraction.  unit fraction - A fraction where the top number (the "numerator") is 1  **numerator** - The top number in a fraction.  **Reciprocal** - The reciprocal of a number is: 1 divided by the number  **Simple interest** - Interest calculated as a percent of the original loan. | **Nurse**  *“We must record how much fluid patient drink when they are recovering from surgery. So, for example, if we give the patient a 300ml glass of orange juice and they only drink 2/3 of it, we have to work out that they have taken in 200ml of fluid.”*  Nurses care for adults who are sick, injured or have physical disabilities.  **Salary: £27,055 to £40,588**  **Sector**: Healthcare |
|  | H4.1 Fractions | **Add, subtract, multiply and divide fractions and mixed numbers.**  **Find the reciprocal of an integer, decimal or fraction.** | When multiplying an integer by a fraction, students may multiply both the numerator and denominator  Students may have difficulty subtracting mixed numbers when the second fraction is bigger than the first |
|  | F4.5 Fractions and decimals | Convert fractions to decimals and vice versa.  Use decimals to find quantities.  Work out divisions with decimal answers.  Write one number as a fraction of another. | Taking the numerator or denominator values of a fraction and usingthem incorrectly as operators in calculations  Not having a secure understanding of conversions between minutes and hours, both ways. |
|  | H4.2 Ratios | **Write ratios in the form 1 : 𝑛 or 𝑛 : 1.**  **Compare ratios.**  **Find quantities using ratios.**  **Solve problems involving ratios.**  **Use bar models to help solve problems** | **Using Mathematics: Real Life Applications**    Ratio is used in many different real-life situations. Converting between different currencies, working out which packet of crisps iis the best value for money, mixing large quantities of cement and scaling up a recipe to cater for more people all involve reasoning using ratio | Students may try to simplify ratios by dividing each side by a different number.  Mixing units in a question | Identifying fractions from visual representations, and writing equivalent fractions.  Finding fractions of quantities.  Division into equal parts. | Trigonometry and sine rule.  Constant of proportionality: integration and solving differential equations. | **NRICH ‘Speeding Boats’**. A good investigation that students could work on in pairs or groups to deepen their understanding of ratio and proportion.  **NRICH ‘Escalator’**. This has similarities to Speeding Boats above and could be used as an additional activity. For both problems students may use a fraction approach rather than ratio so encouraging them to use ratio notation for an alternative solution would be useful.  **NRICH ‘Golden Trail 1’**. A collection of resources that lead students through a variety of examples where the golden ratio can be found.  **NRICH ‘Ratio and Dilutions’**. This is a STEMNRICH activity that provides a practical cross-curricular link to chemistry to help students connect their maths with other subjects. It has an interactive app that allows students to test out their concentration calculations and extends the principle to using two dilutions. Some students may find this quite difficult initially; it could be introduced using the much more simple **NRICH ‘Mixing Lemonade’** activity as a starter. | **Paint technician**  “Every day customers bring me paints to match. I have to understand how changing the ratio of base colours affect the colour of the paint and how to scale the quantities up and down for larger or smaller amount of paint. IF I get it wrong, customers will have patches of different colours and their walls will look quite strange”  Paint sprayers apply coatings and protective finishes to cars, vans, motorbikes and other vehicles.  **Salary: £19,000 to £30,000**  **Sector**: Construction and trades Manufacturing |
|  | H4.3 Ratio and proportion | **Convert between currencies and measures.**  **Recognise and use direct proportion.**  **Solve problems involving ratios and proportion.** | Some students may fail to spot mixed units in ratio conversions.  Students might think that numbers in direct proportion can both be increased/decreased by adding/subtracting the same amount. |
|  | F4.6 Fractions and percentages | Convert percentages to fractions and vice versa.  Write one number as a percentage of another. | **Using Mathematics: Real Life Applications**    Percentages are often used in daily life to express fractions. For example, you might see adverts claiming that 76% of pets prefer a particular brand of food for that 90% of dentist recommend a particular type of toothpaste. Sale price- reductions, discounts and interest rates are usually given as percentages | Students may think 15 as a percentage of 75 | Completing the four rules with integers.  Multiplying and dividing with decimals.  Multiplying and dividing by powers of 10. | Probability. More complex combinations of events are met in Statistics. Often these are best tackled in fraction form, so being able to convert from a probability given as a percentage is useful.  Decision maths. Linear programming may have constraints expressed as percentages of the variables. | **NRICH ‘Matching Fractions, Decimals, Percentages’**. A game similar to pairs that gets students to match equivalent fractions, decimals and percentages.  **NRICH ‘Doughnut Percents’**. A silent group work activity that asks students to match equivalent fractions, decimals and percentages.  **NRICH ‘One or Both’.** A challenging activity where students have to work out how many people took an exam if they know how many people passed both and the percentage of students that passed each exam. | **Statistician**  *“Statistics in the media are often Reported as percentages. This makes it easier to understand by percentages can also be misleading – 600% sounds like a lot but it could just mean 3 out of 5 people interviewed*”  Data analyst-statisticians collect numbers and statistics to identify trends, create models and present results.  **Salary: £23,000 to £62,000**  **Sector**: Science and research |
|  | F4.7 Calculating percentages 1 | Convert percentages to decimals and vice versa.  Find a percentage of a quantity.  Use percentages to solve problems.  **Calculate simple interest** | Students may calculate percentages over 100% incorrectly. |
|  | F4.8 Calculating percentages 2 | Calculate percentage increases and decreases.  **Use percentages in real-life situations.**  **Calculate VAT (value added tax).** | it is a common mistake to forget to add or subtract the percentage change from the original amount, or to give the total amount when you are only asked for the percentage change  Some students may forget to add or subtract in working out, when breaking down a calculation into smaller parts. |
|  | H4.4 Percentages | **Calculate using percentages and ratios.**  **Work out percentage increases and decreases.**  **Solve real-life problems involving percentages.** | Students might forget to add or subtract the percentage change from the original amount.  Some students forget that the amount they are working with in reverse percentage questions is not 100%. |
|  | H 4.5 Fractions, decimals and percentages | **Calculate using fractions, decimals and percentages.**  **Convert a recurring decimal to a fraction.** | Students may multiply by 10 when trying to convert a recurring decimal with two or more recurring digits to a fraction. |
| SUM 1 | **Unit 4: Equations and graphs** | F5.1 Solving equations 1  . | Understand and use inverse operations.  Solve simple linear equations. | **Using Mathematics: Real Life Applications**  Accounting involved a great deal of mathematics. Accountants set up computer spreadsheets to calculate and analyse data. Programs such as Microsoft excel work by applying different equations to values in columns or cells , so you need to know what equations or formulae to use to get the result you need | Students sometimes use a trial and improvement approach rather than creating an equation and using the balancing method to solve it.  Students may think that a negative answer will always be incorrect. | This chapter uses all the manipulation skills worked on in in key stage 3 algebra to help manipulate expressions including factorising and simplifying to solve a variety of equations | Solving forms a large part of the future Mathematics students will study. Students will learn how to manipulate equations containing a larger range of functions to find multiple solutions and have a greater understanding about the number of solutions in a given range.. The ideas surrounding inverses will also be covered in greater detail with students learning about the domain and range of functions as well as the restrictions on these that must sometimes be in place for an inverse to exist. | **NRICH ‘Good Work If You Can Get It’** is a challenging problem in terms of forming and solving. There is a lot of information to get through and students will need to define their own variables for the amount given to each man and the total amount. The trick to solving comes from knowing the fraction of the total amount that each man is due based on the number of days he worked. (nrich.maths.org) | **Recommended Reading**    **The Indisputable Existence of Santa Claus by Hannah Fry and Thomas Oléron Evans**  *Age 11+*  How do you apply game theory to select who should be on your Christmas shopping list? Can you predict Her Majesty's Christmas Message? Will calculations show Santa is getting steadily thinner - shimmying up and down chimneys for a whole night - or fatter - as he tucks into a mince pie and a glass of sherry in billions of houses across the world? Full of diagrams, sketches and graphs, beautiful equations, Markov chains and matrices, this book brightens up the bleak midwinter with stockingfuls of mathematical marvels. Mathematics has never been merrier.    **Seventeen Equations that Changed the World by Professor Ian Stewart (Author)**  From Newton's Law of Gravity to the Black-Scholes model used by bankers to predict the markets, equations, are everywhere -- and they are fundamental to everyday life.Seventeen Equations that Changed the World examines seventeen ground-breaking equations that have altered the course of human history. He explores how Pythagoras's Theorem led to GPS and Satnav; how logarithms are applied in architecture; why imaginary numbers were important in the development of the digital camera, and what is really going on with Schrödinger's cat.  Entertaining, surprising and vastly informative, Seventeen Equations that Changed the World is a highly original exploration -- and explanation -- of life on earth.  **Keywords**  **Equation** - An equation says that two things are equal.  **Solve** - To find a value (or values) we can put in place of a variable that makes the equation true.  **Linear equations** - An equation that makes a straight line when it is graphed.  **Integer** - A number with no fractional part (no decimals).  **Inequality** - An inequality compares two values, showing if one is less than, greater than, or simply not equal to another value  **Formula** - A rule or fact written with mathematical symbols.  **Substitution** - Putting values where the letters are.  **Equation** – An equation says that two things are equal.  **Sequence** - A list of numbers or objects in a special order.  **Term** – In Algebra a term is either a single number or variable, or numbers and variables multiplied together.  **Line segment** - The part of a line that connects two points.  **Midpoint** - The middle of. The point halfway along.  **Perpendicular** - At right angles (90°) to. The symbol is ⊥  **Quadratic equation -** An equation where the highest exponent of the variable (usually "x") is a square (2).  **Parabola** - A special curve, shaped like an arch.  **minimum point** - The smallest value.  **maximum point** - The largest value.  **Trajectory** - The path of an object with mass, such as a kicked ball.  **Asymptotes** - A line that a curve approaches as it heads towards infinity. | **Accountant**  *“Although the computer does the actual calculations, I must insert different equations to tell it what operations to perform and in which order to perform them. It is important to check that the equations are production the correct answers, though.”*  Management accountants look after a company's finances and find ways to improve profitability.  **Salary: £22,000 to £55,000**  **Sector**: Business and finance Managerial |
|  | F5.2 Solving equations 2 | Solve two-step equations. | Some students may use inverses in the wrong order. |
|  | F5.3 Solving equations with brackets | Solve linear equations with brackets.  **Solve equations with unknowns on both sides.** | Some students may leave the second term inside the bracket as it is, or add it to the multiplier  Not understanding that –5x is smaller than –3x. |
|  | H6.1 Linear graphs | **Find the gradient and 𝑦-intercept from a linear equation.**  **Rearrange an equation into the form 𝑦 = 𝑚𝑥 + 𝑐.**  **Compare two graphs from their equations.**  **Plot graphs with equations 𝑎𝑥 + 𝑏𝑦 = 𝑐.** | **Using Mathematics: Real Life Applications**    This is a photograph of the building nicknamed *the Gherkin* in London. The curves and lines of the bui8lding were designed using complex equations and their graphs. Architecture is just one of many professions in which people plot and use graphs in their work | Students may use equations in different forms to compare graphs or find gradient and y-intercept values.  Plotting x and y in the wrong directions. Plotting x and y in the wrong directions. | This chapter provides ample practice at rearranging linear equations, e.g. to turn 2x + 3y = 25 into the form y = mx + c. For questions where graphs are drawn for students, they should be encouraged to check that their rearranged equation is plausible, e.g. the gradient has the correct sign. | n linear programming (using graphs to find the optimal solution to a problem), we draw graphs with ‘constraints’ represented as straight lines. Doing so quickly and accurately is key. These are frequently in the form ax + by = c, rather than y = mx + c.  Finding and using the equations of tangents and normals is a major part of the use of calculus. We frequently know the gradient of the line as well as a single point it passes through, so finding the equation of the line relies on the ideas in this topic.  In A level we make much more use of the tangents to circles, which are straight lines.  The concepts of gradients and coordinates are used heavily in vector methods, which are particularly helpful in solving problems involving straight lines in three dimensions | The computer activity **NRICH ‘Diamond Collector’** provides good practice at working out equations of lines in a game context that requires mathematical creativity.  An unusual use of negative reciprocals is considered in the problem **NRICH ‘Twisting and turning’**  The **NRICH ‘Reflecting Lines’** activity gets students to think about the equation of lines that have been reflected in the y-axis. In particular, it should encourage students to think about what the gradient represents and how a reflection in the vertical axis changes the gradient | **Architect**  *“When designing a new building, I use graphs to help identify and describe the structural properties the building needs to have.”*  Architects design new buildings and the spaces around them, and work on the restoration and conservation of existing buildings.  **Salary: £30,000 to £60,000**  **Sector:** Construction and trades Creative and media |
|  | H6.2 More linear graphs | **Sketch graphs using the gradient and intercepts.**  **Find the equation of a line, given its gradient and one point on the line.**  **Find the gradient of a line through two points.** |
|  | F5.4 Introducing inequalities | Use correct notation to show inclusive and exclusive inequalities.  Show inequalities on a number line.  Write down whole numbers which satisfy an inequality.  **Solve simple linear inequalities.** | **Using Mathematics: Real Life Applications**    Inequalities are one way of showing the ranges of values that have to be met and considered in running a successful business. For example, a business might want wastage to be less than a certain figure, or profit to be great for equal to a particular amount | Some students may forget what is meant by the term integer, or forget that 0 is an integer value.  Using inequality signs incorrectly | This topic relies heavily on skills learnt in previous topics. Most notably, pupils should be secure in their ability to solve and draw linear equations | In Decision Maths, linear programming extends the work on graphing linear inequalities to find optimal solutions given a number of constraints.  Inequalities feature in many A level questions and quadratic inequalities have much greater prominence. | **NRICH ‘Inequalities’** activity uses inequalities in the formulation of a problem involving marbles that can be solved by logical thinking | **Quality Controller**  *“I work in quality control in food standards. One of my jobs is to check that the quality and size of the ingredients match the details shown on the labels”*  Quality control assistants check that a company's services or products meet necessary standards.  **Salary: £20,000 to £30,000**  **Sector**: Administration Engineering and maintenance Manufacturing |
|  | F5.5 More inequalities | **Solve two-sided inequalities.** | Some students may forget to do the same thing to all three parts of a two-sided or double inequality.  Students may forget to change the direction of the inequality sign when multiplying or dividing by a negative number. |
|  | F5.6 Using formulae | Substitute values into formulae and solve equations.  **Change the subject of a formula.**  Know the difference between an expression, an equation and a formula. | **Using Mathematics: Real Life Applications**    Vets use formulae to make sure they are giving animals the correct dosage of medicine for their age and mass. A poodle weighing 6 kg needs a far smaller does of medicine than a 35kg retriever | Applying inverse operations in the wrong order.  Making mistakes with signs when multiplying or dividing with negative numbers. | Since this is a chapter on formulae it has connections with all areas of mathematics where you are required to calculate an answer given several inputs. During this chapter your students will be revisiting the process of calculating areas, perimeters, volumes, solutions to quadratic equations (and looking forward to using trigonometric functions,  Pythagoras’ theorem and equations of straight-line graphs). | Students looking further ahead will need to be fluent in their use of all the formulae mentioned in this chapter, particularly those relating to equations of motion and calculus topics. A level students will be required to rearrange formulae with the variable appearing several times, for example, rearrange a formula containing a quadratic term, say x2 , to make x the subject. Further to this, recurrence relations will be tied together with different types of sequences where students will be required to recognise more than one way of rearranging the subject to give a recurrence relation to employ Numerical Methods. | The activity NRICH **‘How Do You React?’** could link this section with Interpretation of graphs. The students are required to come up with their own formula for this situation  Links can be made forward with the activity NRICH **‘Making Maths: Make a Pendulum’** and an investigation, where you drop an object from varying heights and find the time taken to fall. This data can then be plotted and students could come up with a formula to describe this situation | **Veterinary surgeon**  *“I need to make sure I give the animals I treat the correct amount of medicine. I do this by using formulae that take into account their age, mass and the ratio between prescribed medicine”*  Vets diagnose and treat sick or injured animals.  **Salary: £30,000 to £50,000**  **Sector**: Science and research Animal care |
|  | H6.3 Graphing rates of change | **Draw and interpret distance–time graphs.**  **Calculate average speed from a distance–time graph.**  **Understand velocity–time graphs.**  **Find acceleration and distance from velocity–time graphs.** | **Using Mathematics: Real Life Applications**    All sorts of information can be obtained from graphs in real-life context. The shape of a graph, its gradient and the area underneath it can tell use about speed, time, acceleration, prices, earnings, break-even points or the values of on currency against another, among other things. | Students may not plot lines and points exactly when drawing graphs.  Some students may ignore errors on a graph. | Features of straight lines, such as the concept of gradient, were discussed in Straight-line graphs.  Proportional methods from Proportion are used, e.g. ‘how many Indian rupees would you get for US$600?’  Methods from Mensuration are important, including working with time, e.g. calculating speed in miles per hour when a distance of 15 miles is covered in 20 minutes.  Practice in finding areas, in particular by breaking down compound shapes into rectangles, triangles and trapezia. | Differentiation, finding the gradient at a point on a curve.  Integration, finding the area under curves.  Equations of motion.  Regression, fitting a (often linear) model to a set of data. | If any members of staff have recently run a marathon, their split times (usually downloadable after the event) could be used as the source of a distance-time graph. When did they ‘hit the wall’? | **Racing driver**  *“My car needs to perform at its optimum limits. We generate and analyse diagnostic graphs to calculate the slight changes that would increase power, acceleration and top speed.”*  Motorsport engineers design, build and test racing cars and bikes.  Salary: £22,000 to £60,000  **Sector**: Engineering and maintenance Manufacturing Sports and leisure |
|  | H6.4 Real-life graphs | **Draw and interpret real-life linear graphs.**  **Recognise direct proportion.**  **Draw and use a line of best fit.** | Students may not completely answer the question. |
|  | F5.7 Generating sequences | Recognise and extend sequences. | **Using Mathematics: Real Life Applications**    Finding a pattern and working out how the parts of the pattern fit together is important in scientific discovery. Scientists use sequences to model and solve real- life problems, such as estimating how quickly disease spread. | Some students struggle with decreasing sequences. | This topic provides a good opportunity to return to work on basic calculations and properties of numbers. There are opportunities to consider how square and cube numbers are related to physical shapes and how a numerical sequence can be linked to geometrical or physical patterns. | This topic will be built upon in KS5. Having a strong understanding of this concept will be necessary for students to extend their knowledge in A level modules. In addition to the notation learnt at GCSE, students will also learn to describe sequences as arithmetic progressions. They will calculate terms in a sequence and the sum to n terms.  In addition to arithmetic progressions, geometric progressions are a further extension of this topic, which builds upon sequences such as 2, 4, 8, 16, 32. Students will learn to calculate terms in a sequence and the sum to n terms, including the sum to infinity for converging sequences. | **NRICH ‘Seven Squares’ task**. The teacher notes with this resource suggest how to use this in the classroom, enabling the formulation of the position-to-term rule to naturally emerge from discussions on the construction of the sequence. (nrich.maths.org)  Search for any of the ‘Zeno’s Paradox’ videos on the internet  **NRICH ‘Fibs’ task.** (nrich.maths.org)  **NRICH ‘Steel Cables’** task. (nrich.maths.org)  **NRICH ‘Handshakes’** task. (nrich.maths.org)  **NRICH ‘Mystic Rose’** is a different form of the ‘Handshakes’ problem. (nrich.maths.org) | **Medical researcher**  *“When a new outbreak of a disease occurs need to work out how quickly it is spreading. To do this I look at the sequence in which the numbers of victims are increasing. I use the sequence to predict how many people will become infected in a certain length of time.”*  Research scientists plan and lead experiments and investigations on a range of scientific topics.  **Salary: £17,688 to £43,000**  **Sector:** Environment and land Science and research |
|  | F5.8 Using the 𝑛th term of a sequence | **Use the nth term to generate terms of a sequence.**  **Find the nth term of an arithmetic sequence.** | Some students forget how the common difference relates to the nth term, |
|  | H6.5 Line segments | **Find the coordinates of the midpoint of a line segment.**  **Find the gradient and length of a line segment.**  **Find the equations of lines parallel or perpendicular to a given line.** | **Using Mathematics: Real Life Applications**    Graphs are used to process information, make predictions, and generalise patterns from sets of data. The nature of the data and the relationship between values reveals the shape and form of the graph | Students may mix up x- and y-coordinates when using the formula to find a midpoint.  Some students may make errors when deciding whether a gradient is positive or negative. | This links back right back to Properties of numbers Introduction to algebra and Functions and sequences for the basic manipulation of numbers and algebra.  There are also strong connections with Straight-line graphs. | A level mathematics further develops the connections between algebra and geometry. Students who go on to A level will be expected to be fluent in moving between descriptions of the function as a graph and as an equation, and will further their knowledge of circles not centred on the origin. Further to this they will need to answer questions of the type: given two graphs y = f(x) and y = g(x) on the same pair of axes, what do the number of intersections mean for the solutions of the equation f(x) = g(x)?  From an application point of view, simple harmonic motion, projectiles and parabolic motion are studied and have foundations in this chapter | The activity **NRICH ‘Parabolic Patterns’** allows for the use of GeoGebra or a graphical calculator to give the students a chance to consolidate their recently gained knowledge of identifying an equation of a parabola. For Higher students you can increase the difficulty by attempting the activity **NRICH ‘More Parabolic Patterns’**.  As an introduction to the topic of reciprocal graphs this **NRICH ‘More Realistic Electric Kettle’** activity could be used. This activity also has connection to a task that could happen in the students’ science lessons and offers an opportunity to combine such a lesson. The students get a chance to plot a curve and find a way to figure out the equation of the line  The **NRICH ‘Guessing the Graph’** activity can be used for students to make their own data, plot the data and use their knowledge of the graphs of different functions to suggest possible curves to fit to the data.  This **NRICH ‘What’s That Graph?’** activity can be used to draw connections between the physical world and the mathematics the students have covered in this chapter. | **Geologist**  *“I study the earth using gravity, magnetic, electrical and seismic methods. I used this graph in a study of the pacific and Atlantic Oceans. I need to be able to understand equations and recognise the features of graphs to understand and interpret it.”*  Seismologists study shock waves created by earthquakes and volcanic activity. They also work in oil, gas and minerals exploration.  **Salary: £20,000 to £50,000**  **Sector**: Science and research Environment and land |
|  | H6.6 Quadratic graphs | **Draw quadratic graphs.**  **Solve quadratic equations using graphs.**  **Identify the line of symmetry of a quadratic graph.**  **Interpret quadratic graphs relating to real-life situations.** | Students may fail to spot a negative gradient. |
|  | H6.7 Cubic and reciprocal graphs | **Draw graphs of cubic functions.**  **Solve cubic equations using graphs.**  **Draw graphs of reciprocal functions.**  **Recognise a graph from its shape.** | Students may not use all the information in an equation to match a graph to its equation |
|  | H6.8 More graphs | **Interpret linear and non-linear real-life graphs.**  **Draw the graph of a circle.** | Understand and be able to define the meaning of correlation.  Manipulation of surds. |
| SUM 2 | **Unit 5: Angles and trigonometry** | F6.1 Properties of shapes | Solve geometric problems using side and angle properties of quadrilaterals.  Identify congruent shapes. | **Using Mathematics: Real Life Applications**    Many people use geometry in their jobs and daily lives. Artists, craftspeople, builders, designers, architects and engineers use shape and space in their jobs, but almost everyone uses lines, angles a, patterns and shapes in different ways everyday | Missing lines of symmetry or thinking parallelograms have lines of symmetry. | This topic provides ample practice for mental methods of addition and subtraction.  Students could also practise measuring lengths and angles by checking the precision of some hand drawn shapes: is a shape with four sides varying between 6.9–7.1 cm and 89–91° really a square? | In calculus students will learn to use perpendicular and parallel lines when working out the equations of tangents and normals. Later in calculus, students will learn to calculate the volumes of solids created by revolving a curve around an axis through 360°.  Students will learn to classify functions as ‘odd’ or ‘even’ based on the symmetry properties of their graphs. | Quadrilateral Rummy, in the ATM book **Geometry Games**. A variation is NRICH **‘Quadrilaterals Game’.**  **NRICH ‘Quadrilaterals’** asks students to find as many different quadrilaterals as possible by joining dots on the circumference of a circle. The problem initially starts with a circle with eight evenly spaced dots, but is easily adapted for other numbers of dots. This problem can be used with students of all levels to revisit vocabulary, then using the angles of isosceles triangles to work out the angles in each quadrilateral and start to explore the properties of cyclic quadrilaterals (Higher only). A slightly more structured version of this activity is **NRICH ‘Cyclic Quadrilaterals’**. (nrich.maths.org) | **Recommended Reading**    **Flatterland by Ian Stewart**  *Age 13+*  In 1884, Edwin A. Abbott published “Flatland”; a brilliant novel about mathematics and philosophy that charmed and fascinated all of England.  Now, Ian Stewart has written a fascinating, modern sequel to Abbott's book. Through larger-than-life characters and an inspired story line, "Flatterland" explores our present understanding of the shape and origins of the universe, the nature of space, time, and matter, as well as modern geometries and their applications.    **Mathematics and the Physical World by Morris Kline**  *Age 13+*  A stimulating account of development of basic mathematics from arithmetic, algebra, geometry and trigonometry, to calculus, differential equations and non-Euclidean geometries. Also describes how maths is used in optics, astronomy, motion under the law of gravitation, acoustics, electromagnetism, and other aspects of physics.    **Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace by Leonard Mlodinow**  *Age 14+*  Anyone who thought geometry was boring or dry should prepare to be amazed. Despite its worthy cover this book is exactly what its title says - a story - and the plot of this story involves life, death and revolutions of understanding and belief. It stars the some of the most famous names in history, from Euclid who laid the logical foundations, to Albert Einstein, who united space and time in a single non-Euclidean geometry. It offers an alternative history of mathematics, revealing how simple questions anyone might ask about space - in the living room or in some other galaxy - have been the hidden engines of the highest achievements in science and technology.  **Keywords**  **Congruent** - The same shape and size (but we are allowed to flip, slide or turn).  **Similar** - When one shape can become another after a resize, flip, slide or turn.  **Diagonal** – A line segment that goes from one corner to another, but is not an edge.  **Bisect** - To divide into two equal parts.  **Parallel lines** - Lines on a plane that never meet. They are always the same distance apart  **alternate angles** - When two lines are crossed by another line (the Transversal), a pair of angles, on the inner side of each of those two lines, but on opposite sides of the transversal are called Alternate Angles.  **corresponding angles** - When two lines are crossed by another line (which is called the Transversal), the angles in matching corners are called corresponding angles  **Exterior angle** - The angle between any side of a shape, and a line extended from the next side.  **interior angle** - An angle inside a shape, between two joined sides.  **Polygon** - A plane shape (two-dimensional) with straight sides.  **regular polygon** - A polygon is regular when all angles are equal and all sides are equal | **Civil Engineer**  *“I use CAD package to plot lines and angles and show the direction of traffic flow when I design new road junctions”*  Civil engineers design and manage construction projects, from bridges and buildings to transport links and sports stadiums.  **Salary: £30,000 to £70,000**  **Sector**: Construction and trades Engineering and maintenance |
|  | F6.2 Angles in parallel lines | Understand and use the angle properties of parallel lines.  Find missing angles using corresponding and alternate angles. | **Using Mathematics: Real Life Applications**    People who work in many varied and unrelated jobs rely on an understanding of angles and how shapes fit with others in their daily work. These include designers, architect, opticians, and tree surgeons among others | Failing to see Z-shapes or F-shapes and so not deducing the location of alternate or corresponding angles correctly, particularly when there are two or more diagonals across the parallel lines | This connects directly to Shapes and solids, in which students learned about properties of shapes and that interior angles of a triangle sum to 180°. It also uses the concepts learned in Equations regarding the solving of equations, in particular linear equations. | These basic facts are still used in problem-solving at A level. ‘Vertically opposite angles are equal’ and ‘the sum of angles on a straight line is 180°’ are both used in vectors work, and numerous basic facts and circle theorems are used in coordinate geometry problems on the (x, y) plane. In addition, students will extend their knowledge of complementary angles in Trigonometry and Complex Numbers in order to solve equations and correctly use Argand diagrams | NRICH **‘Notes on a Triangle’** uses a beautiful René Jodoin film as a prompt for students to think about a whole host of properties of triangles and other shapes.  **NRICH ‘Terminology’**. This problem involves using angle properties of an equilateral and isosceles triangle, as well as angles along a straight line, to form some algebra  In **NRICH ‘Cyclic Quadrilaterals’**, the first part of each of the questions requires students to draw isosceles triangles on different dotty circles, and work out the angles  Tessellations make an interesting investigation and students could now think about how angle facts can help them know whether a regular shape tessellates or not.  **NRICH ‘Semi-regular Tessellations’** has some interesting additions to the problem, tessellating two or more regular polygons together. It also has a very good interactive program for testing ideas.  **‘Simple Quadrilaterals Tessellate the Plane’** is another interesting investigation in why we can tessellate any quadrilateral. (cut-the-knot.org)  For students who enjoy tessellating you may wish to extend them by looking at Penrose tilings and further consolidating their knowledge of angles. | **Structural Engineer**  *“I had to work quite carefully with the 360 degrees around the centre to play each of the 32 pods correctly on the London Eye”*  Structural engineers help to design and build large structures and buildings, like hospitals, sports stadiums and bridges.  **Salary: £28,000 to £50,000**  **Sector**: Construction and trades Engineering and maintenance |
|  | F6.3 Angles in triangles | Solve angle problems in triangles.  Understand angle proofs about triangles. | Incorrectly identifying which two angles are equal in an isosceles triangle  Confusing letters when deriving proofs. |
|  | H5.1 Angle properties of triangles and quadrilaterals | **Derive and use the sum of angles in a triangle and in a quadrilateral.**  **Derive and use the fact that the exterior angle of a triangle is equal to the sum of the two opposite interior angles.** | Some students may find it difficult to see shapes that are made or to identify when lines are parallel or the same length. |
|  | F6.4 Exterior and interior angles | **Calculate the interior and exterior angles of regular polygons.** | Drawing exterior angles incorrectly  Tell students to imagine an ant walking round the shape.  Many students find multi-step problems very challenging. |
|  | F6.5 More exterior and interior angles | **calculate the interior and exterior angles of polygons.**  **Explain why some polygons fit together and others do not.** | Failing to spot isosceles triangles in diagrams  Difficulty identifying appropriate reasons to accompany each stage of working or writing these reasons correctly. |
|  | H5.2 Interior angles of a polygon | **Calculate the sum of the interior angles of a polygon.**  **Use the interior angles of polygons to solve problems.**  **Use 𝑥 for the unknown to help you solve problems.** | Students may fail to subtract 2 from n before finding the sum of interior angles. |
|  | H5.3 Exterior angles of a polygon | **Know the sum of the exterior angles of a polygon.**  **Use the angles of polygons to solve problems.** | Students may not spot relationships between angles within diagrams |
|  | F6.6 Geometrical problems | Solve angle problems using equations.  Solve geometrical problems showing reasoning.  Use 𝑥 for the unknown to help you solve problems. | Not knowing how to get started on a problem |
|  | H5.4 Pythagoras’ theorem 1 | **Calculate the length of the hypotenuse in a right-angled triangle.**  **Solve problems using Pythagoras’ theorem.** | **Using Mathematics: Real Life Applications**    Builders, carpenters, garden designers and navigators all use Pythagoras’ theorem in their jobs. It is a method based on right angled triangles that helps them to work out unknow lengths. | Students may not find the square root of the sum of the smaller sides squared. | The ‘standard’ diagram to demonstrate Pythagoras’ theorem shows three squares on the sides of the right-angled triangle. If these squares were replaced with semicircles, does it still work? Is the area of a semicircle on the hypotenuse equal to the sums of the semicircles on the other two sides? By exploring this problem you can link back to work on area of circle sectors in Area. There are also many opportunities that are used in this chapter to explore area and perimeter problems for regular and composite shapes now pupils are better equipped with skills to find missing lengths associated with right-angled triangles | At A level, Pythagoras’ theorem will be employed in many different ways. The most obvious of those is in coordinate geometry where pupils will use Pythagoras’ theorem to find the length of line segments in 2D and 3D as well as finding the radius of a circle given the centre and a point on the curve and deducing the equation of the circle. Less direct is its application to deriving trigonometric identities, notably: and its use in deriving the compound angle formulae.  It will also be used to calculate the magnitude of vectors in 2D and 3D, and hence many other values associated with compound measures | **NRICH ‘Tilted Squares’** has interactive resources available to explore tilted squares and methods for calculating their area in detail that may be of use to you. Students sometimes struggle with the idea of a tilted square and the supporting animations make the idea very clear. Using dotted squared paper rather than standard squared paper may also make things easier. These tilted squares can be used to derive and prove Pythagoras’ theorem in different ways should you wish to extend the most able students.  Once pupils are confident using Pythagoras’ theorem an appropriate extension might be to consider different ways in which the theorem can be proved. There are multiple ways the theorem can be proved and you can search for these proofs online. For example, **NRICH ‘Pythagoras Proofs’**, the first of which comes from the tilted squares problem  **NRICH ‘Nicely Similar’** is a problem students will have to think carefully about what information they have been given and label the lengths of the two right-angled triangles in their sketch carefully. They will need to compare the three triangles they know and appropriately form and solve the equations to find the missing lengths. This problem can be used to revise similarity.  **NRICH ‘Liethagoras’ activity**: in which an alternative theorem based on right-angled triangles is offered for students  to prove or disprove | **Navigation officer**  *“I use Pythagoras’ theorem to help me navigate the ship… I need to know how far away we are from port and I use the theorem to calculate this…”*  Merchant Navy deck officers look after the day-to-day running and navigation of ships, and take care of passengers and cargo.  **Salary: £12,700 to £40,000**  **Sector**: Transport Emergency and uniform services |
|  | 5.5 Pythagoras’ theorem 2 | **Calculate the length of a shorter side in a right-angled triangle.**  **Solve problems using Pythagoras’ theorem.** | Students may assume that the unknown length in a triangle is always the hypotenuse. |
|  | H5.6 Trigonometry 1 | **Use trigonometric ratios to find lengths in a right-angled triangle.**  **Use trigonometric ratios to solve problems.**  **Find angles of elevation and angles of depression.** | **Using Mathematics: Real Life Applications**    Trigonometry means ‘triangles measurements’ and is very useful for finding leghts of sides and sizes of angles of triangles. Trigonometry is used to work out lengths and angles in navigation, surveying, astronomy, engineering, construction and even in the placements of satellites and satellite recievers | Students may not rearrange the equation correctly when the unknown is the denominator of the fraction. | There are many opportunities to combine previous concepts covered at key stage 3 and connect geometric ideas. Opportunities should be taken to look at general problems based on right-angled triangles to encourage students to identify the different methods available to them in solving and what information is required to apply each one.. | Trigonometry forms a large part of the A level syllabus. Alongside the 2D and 3D problem-solving applications, students will explore the domain and range of each function in detail considering the features of each one. Students will learn to solve more complicated equations involving trigonometric functions and list all solutions in a possible domain. Knowledge of identities will be used to manipulate expressions and reciprocal functions will be used to extend these ideas. | **NRICH ‘Where Is the Dot?’** is an animation that could be used to explore what the trig functions store for angles greater than 90°.  Students can make their own clinometers using **NRICH ‘Making Maths: Clinometer’**  **MEP: Trigonometry Unit 4: Activity 4.3 Clinometers** | **Geologist**  ***“****I use a theodolite to work out the height of mountains. You basically point it at the top of the mountain. The theodolite uses the principles of trigonometry to measure the angles and the distance”*  Geoscientists study the Earth's structure and formation, and analyse rocks to explore its natural mineral and energy resources.  **Salary: £28,000 to £42,000**  **Sector**: Environment and land Science and research |
|  | H5.7 Trigonometry 2 | **Use trigonometric ratios to calculate an angle in a right-angled triangle.**  **Use trigonometric ratios to solve problems.**  **Know the exact values of the sine, cosine and tangent of some angles.** | Students may fail to use the inverse function on the calculator. |